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10MAT41

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer any FIVE full questions, selecting atleast TWO questions from each part.***
2. Use of statistical table is permitted.

PART – A

1 a. Employ Taylor's series method to find an approximate solution to find y at x = 0.1 given $\frac{dy}{dx} = x - y^2$, y(0) = 1 by considering upto fourth degree term. **(06 Marks)**

b. Solve the following by Euler's modified method $\frac{dy}{dx} = \log(x+y)$, y(0) = 2 to find y(0.4) by taking h = 0.2. **(07 Marks)**

c. Given $\frac{dy}{dx} = x^2(Hy)$ and y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979. Evaluate y(1.4) by Adams-Bash forth method. Apply corrector formula twice. **(07 Marks)**

2 a. Solve $\frac{dy}{dx} = 1 + xz$ and $\frac{dz}{dx} = -xy$ for x = 0.3 by applying Runge Kutta method given y(0) = 0 and z(0) = 1. Take h = 0.3. **(06 Marks)**

b. Use Picard's method to obtain the second approximation to the solution of $\frac{d^2y}{dx^2} - x^3\frac{dy}{dx} - x^3y = 0$ given y(0) = 1, y'(0) = 0.5. Also find y(0.1). **(07 Marks)**

c. Apply Milne's method to compute y(0.4) given y'' + xy' + y = 0, y(0) = 1, y'(0) = 0, y(0.1) = 0.995, y'(0.1) = -0.0995, y(0.2) = 0.9802, y'(0.2) = -0.196, y(0.3) = 0.956 and y'(0.3) = -0.2863. **(07 Marks)**

3 a. Derive Cauchy-Riemann equation in Cartesian form. **(06 Marks)**

b. Find an analytic function f(z) whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$ and hence find its imaginary part. **(07 Marks)**

c. If f(z) is a holomorphic function of z, then show that $\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2$. **(07 Marks)**

4 a. Discuss the transformation $w = z + \frac{1}{z}$. **(06 Marks)**

b. Find the BLT which maps the points z = 1, i, -1 to w = i, 0, -i. Find image of $|z| < 1$. **(07 Marks)**

c. Evaluate $\int_C \left\{\frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}\right\} dz$ where 'C' is circle $|z| = 3$. **(07 Marks)**

PART – B

5 a. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ interms of Legendre polynomials. **(06 Marks)**

b. Obtain the solution of $x^2y'' + xy' + (x^2 - x^2)\,y = 0$ interms of $J_n(x)$ and $J_{-n}(x)$. **(07 Marks)**

c. Derive Rodrique's formula $P_n(x) = \frac{1}{2^n \lfloor n} \frac{d}{dx^n}[(x^2-1)^x]$. **(07 Marks)**

6 a. State the axioms of probability. For any two events A and B, prove that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. **(06 Marks)**

b. A box 'A' contains 2 white and 4 black balls. Another box 'B' contains 5 white and 7 black balls. A ball is transferred from the box A to the box B. Then a ball is drawn from the box B. Find the probability that it is white. **(07 Marks)**

c. In a certain college 4% of the boys and 1% of girls are taller than 1.8m. Further more 60% of the students are girls. If a student is selected at random and is found to be taller than 1.8m, what is the probability that the student is a girl? **(07 Marks)**

7 a. The probability density of a continuous random variable is given by $p(x) = y_0 e^{-|x|}, -10 < x < \infty$. Find y_0. Also find the mean. **(06 Marks)**

b. Obtain the mean and variance of binomial distribution. **(07 Marks)**

c. In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and SD of 60 hours. Estimate the number of bulbs likely to burn for.

i) More than 2150 hours.

ii) Less than 1950 hours.

iii) More than 1920 hours but less than 2160 hours.

Given A(1.5) = 0.4332, A(1.83) = 0.4664, A(2) = 0.4772. **(07 Marks)**

8 a. In a city 'A' 20% of a random sample of 900 school boys had a certain slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions is significant? Why? **(06 Marks)**

b. A manufacturer claimed that atleast 95% of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 of them were faulty. Test his claim at a significance level of 1% and 5%. **(07 Marks)**

c. A set of five similar coins is tossed 320 times and the result is

No. of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution $\left[\chi^2_{0.05} = 11.07 \text{ for } 5df\right]$. **(07 Marks)**

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10ES42

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Microcontrollers

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART - A

1 a. With neat diagram, explain the internal architecture of 8051. **(10 Marks)**

b. Compare microprocessor with microcontroller. **(06 Marks)**

c. Calculate the time required for 2 machine cycle instruction
i) 12MHz ii) 11.0592MHz. **(04 Marks)**

2 a. Explain following instructions mentioning their addressing mode and byte size.
i) XCHD A, @R_0 ii) MOVC A, @ A + DPTR iii) SUBB A, # 55h
iv) DA A v) JBC bit, rel vi) ORL C, 100h. **(12 Marks)**

b. Explain the operation of following code with respect to stack.
MOV SP, # 10h
PUSH SP
POP 0E0h
ADD A, # 10h. **(05 Marks)**

c. Briefly explain the range of relative addressing. **(03 Marks)**

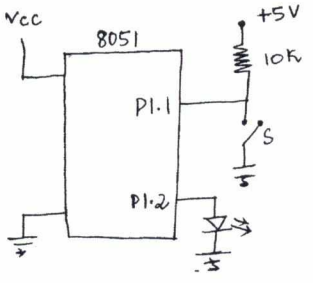
3 a. Explain ORG, END, DB and EQU directives. **(04 Marks)**

b. Using subroutine, write a subroutine to get one second delay. Use subroutine names as follows :
for 1 ms subroutine : ONE _MILI_SUB
for 1s Subroutine : ONE_SEC_SUB. **(10 Marks)**

c. Write a program to add 5 numbers. Numbers are stored between internal RAM 60h to 64h. Store the result in R_0 & A. **(06 Marks)**

4 a. With neat diagram, explain the internal structure of P1.0. **(07 Marks)**

b. Write a program to read switch as shown below. If switch is closed, turn ON the LED else turn OFF the LED. **(05 Marks)**



c. With interfacing diagram, write a program to rotate a stepper motor clockwise. **(08 Marks)**

PART - B

5 a. Explain TMOD & TCON registers with its bit pattern. **(08 Marks)**

b. What are interrupts? Specify vector location or interrupts in 8051. **(04 Marks)**

c. Generate square wave of 5kHz & 10kHz on P1.2 & P1.3. Continuously use timer 1 and time 2 in mode 2 for the purpose. **(08 Marks)**

6 a. Write the steps required to transfer data serially on 8051. **(08 Marks)**
b. Write a program to send 'Y' serially on 8051. Use baud rate of 2400 bauds. **(06 Marks)**
c. Briefly show control word of 8255 & specify mode selection. **(06 Marks)**

7 a. Explain the internal registers of MSP 430. **(10 Marks)**
b. Explain status register of MSP 430. **(10 Marks)**

8 a. Explain different addressing modes supported by MSP 430. **(12 Marks)**
b. Briefly explain the clock system of MSP 430. **(08 Marks)**

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10ES43

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Control System

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Draw the F-V the F-I analogous circuits for the mechanical system shown in Fig.Q.1(a) with necessary equations. **(12 Marks)**

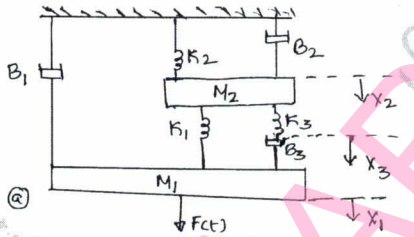


Fig.Q.1(a)

b. For the rotational mechanical system shown draw the torque-voltage analogous circuit for Fig.Q.1(b). **(08 Marks)**

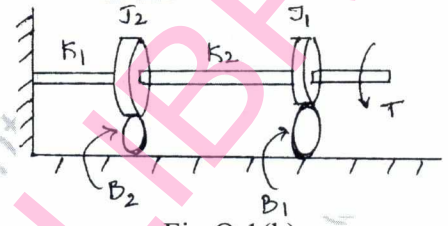


Fig.Q.1(b)

2 a. Using the block diagram reduction technique find $\frac{C}{R}$ for Fig.Q.2(a). **(05 Marks)**

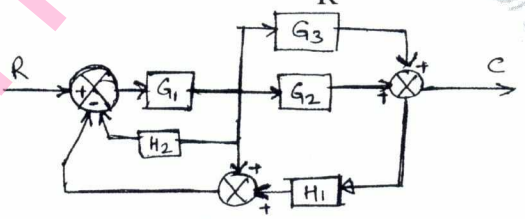


Fig.Q.2(a)

b. Draw the signal flow graph for the block diagram shown in Fig.Q.2(b) and find the TF. **(08 Marks)**

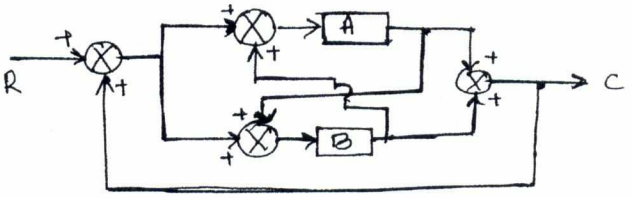


Fig.Q.2(b)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

c. Draw the signal flow graph and find the TF Fig.Q.2(c). **(07 Marks)**

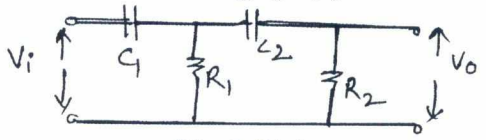


Fig.Q.2(c)

3 a. Find the error coefficients K_p, K_v and K_a for the system having $G(s) = \frac{10}{s^2 + 2s + 9}$ & $H(s) = 0.2$. **(06 Marks)**

b. Find K_1 so that $\xi = 0.35$. Find the corresponding time domain specifications for Fig.Q.3(b). **(06 Marks)**

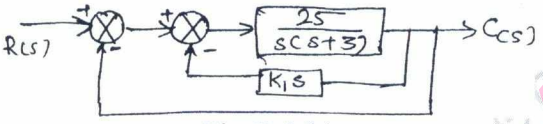


Fig.Q.3(b)

c. With respect to a second order system define the following by drawing neat response curve and expressions: i) Maximum overshoot (M_p); ii) Time delay (t_d); iii) Time constant (T); iv) Rise time (t_r). **(08 Marks)**

4 a. What are the necessary and sufficient conditions for a system to be stable according to Routh-Hurwitz criterion? **(04 Marks)**

b. What value of K makes the following unity feedback system stable?

$G(s) = \frac{K(s+1)^2}{s^3}$. **(04 Marks)**

c. Find how many roots have real parts greater than -1 for the characteristic equation. $s^3 + 7s^2 + 25s + 39 = 0$. **(04 Marks)**

d. How many roots of the characteristic polynomial lie in the right half of S-plane, the left half of s-plane and on jw axis. Comment on the stability of the system. $P(s) = s^5 + 2s^4 + 2s^3 + 4s^2 + s + 2$. **(08 Marks)**

PART – B

5 a. What are the angle and magnitude conditions that a point on root locus has to satisfy? **(06 Marks)**

b. Sketch the root locus for the unity feedback control system whose open loop transfer function is $G(s) = \frac{1}{s(s+2)(s^2+4s+13)}$. **(14 Marks)**

6 a. With respect to Nyquist criterion explain the following:

i) Encirclement of a point.
ii) Analytic function and its singularities.
iii) Mapping theorem or principle of argument.
iv) Find the number of encirclements of point A in Fig.Q.6.1(a) and 6.1(b). **(08 Marks)**

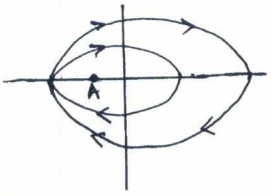


Fig.Q.6.1(a)

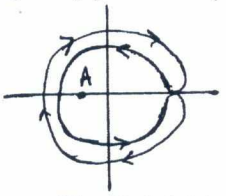


Fig.Q.6.1(b)

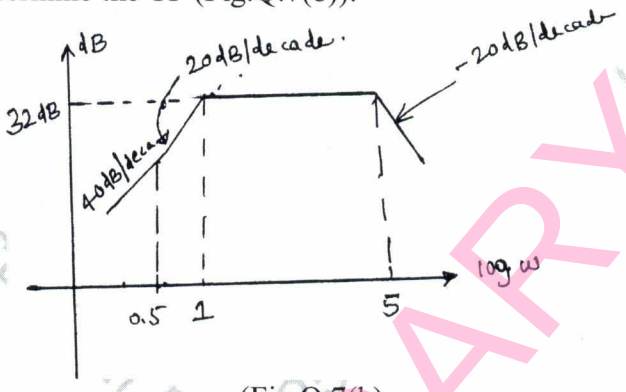
b. For the open loop TF of a feedback control system $G(s)H(s) = \frac{K(1+2s)}{s(1+s)(1+s+s^2)}$. Sketch the complete Nyquist plot and hence find the range of K for stability using Nyquist criterion. **(12 Marks)**

7 a. Draw the bode plot for a system having

$$G(s) = \frac{K(1+0.2s)(1+0.025s)}{s^3(1+0.01s)(1+0.005s)}.$$

Comment on the stability of the system. Also find the range of K for stability. **(12 Marks)**

b. For the plot shown determine the TF (Fig.Q.7(b)). **(08 Marks)**



(Fig.Q.7(b)

8 a. What are the advantages of state space analysis? **(04 Marks)**

b. A system is described by the differential equation

$$\frac{d^3y}{dt^3} + \frac{3d^2y}{dt^2} + \frac{17dy}{dt} + 5y = 10u(t)$$

Where y is the output and u is the input to the system. Determine the state space representation of the system. **(06 Marks)**

c. Obtain the state equations for the electrical network shown in Fig.Q.8(c). **(10 Marks)**

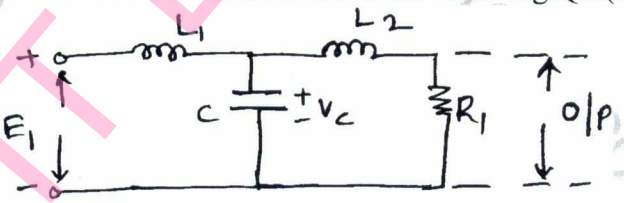


Fig.Q.8(c)

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10EC44

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Signals & Systems

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Define signal and system with example. And briefly explain operations performed on independent variable of the signal. **(06 Marks)**

b. Determine whether the following signal is energy signal or power signal and calculate its energy or power

$$x(t) = \text{rect}\left(\frac{t}{T_0}\right)\cos\omega_0 t.$$ **(04 Marks)**

c. Find whether the following system is stable, memory less, linear and time invariant?

$$y(t) = \sin[x(t+2)]$$ **(04 Marks)**

d. Two signals x(t) and g(t) as shown in Fig. Q1 (d). Express the signals x(t) interms of g(t). **(06 Marks)**

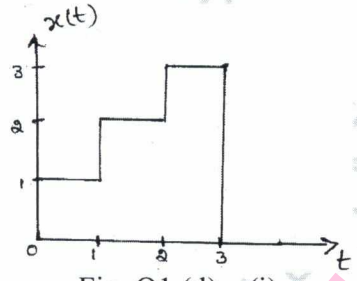

Fig. Q1 (d) – (i)

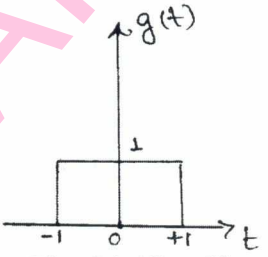

Fig. Q1 (d) – (ii)

2 a. Given the signal x(t) as shown in Fig. Q2 (a). Sketch the following:

i) $x(-2t+3)$ ii) $x\left(\frac{t}{2}-2\right)$ **(04 Marks)**

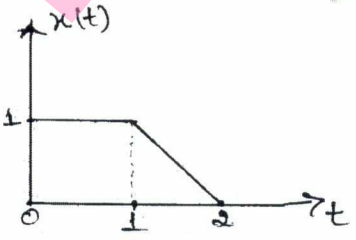

Fig. Q2 (a)

b. For a DT LTI system to be stable show that,

$$S \overset{\Delta}{\equiv} \sum_{K=-\infty}^{K=+\infty} |h(K)| < \infty$$ **(05 Marks)**

c. Two discrete time LTI systems are connected in cascade as shown in Fig. Q2 (c). Determine the unit sample response of this cascade connection. **(06 Marks)**

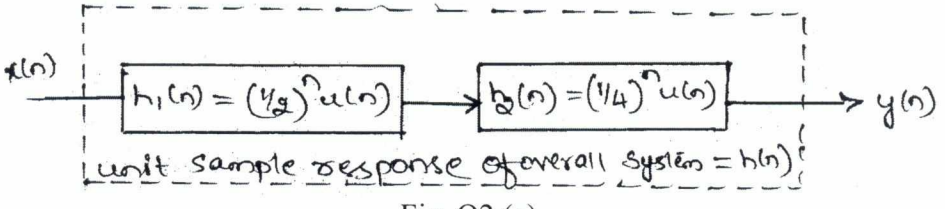

Fig. Q2 (c)

d. Find convolution of 2 finite duration sequences,
$h(n) = a^n u(n)$ for all n and
$x(n) = b^n u(n)$ for all n
i) when $a \neq b$ ii) when $a = b$ **(05 Marks)**

3 a. Determine the LTI systems characterized by impulse response.
i) $h(n) = n\left(\frac{1}{2}\right)^n u(n)$ ii) $h(t) = e^{-t}u(t+100)$
Stable and causal. **(06 Marks)**

b. Find the forced response of the following system:
$y(n) - \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = x(n) + x(n-1)$ for $x(n) = \left(\frac{1}{8}\right)^n u(n)$. **(08 Marks)**

c. Draw direct form II implementation for the system described by the following equation and indicate number of delay elements, adders, multipliers.
$y(n) - 0.25y(n-1) - 0.125y(n-2) - x(n) - x(n-2) = 0$ **(06 Marks)**

4 a. Prove the following properties of DTFS:
i) Convolution in time. ii) Modulation theorem. **(06 Marks)**

b. Determine the complex exponential Fourier series for periodic rectangular pulse train shown in Fig. Q4 (b). Plot its magnitude and phase spectrum. **(08 Marks)**

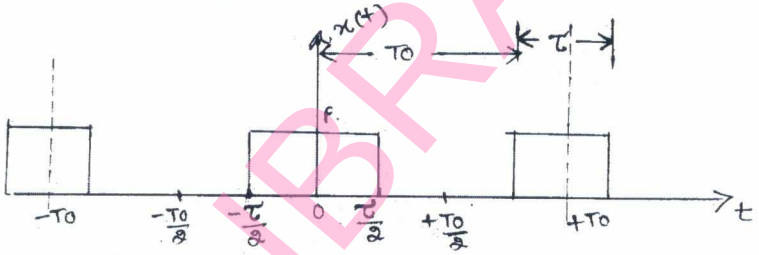

Fig. Q4 (b)

c. Determine the DTFS representation for the signal $x(n) = \cos\left(\frac{n\pi}{3}\right)$. Plot the spectrum of x(n). **(06 Marks)**

PART – B

5 a. State and prove the following properties of DTFT:
i) Parsevai's theorem ii) Linearity **(06 Marks)**

b. Find the DTFT of the signals shown,
i) $x(n) = \left(\frac{1}{4}\right)^n u(n+4)$ ii) $x(n) = u(n)$ **(08 Marks)**

c. Find the inverse Fourier transform of the rectangular spectrum shown, **(06 Marks)**

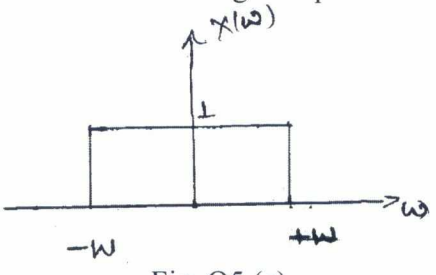

Fig. Q5 (c)

6 a. Consider the continuous time LTI system described by,

$$\frac{d}{dt}y(t)+2y(t)=x(t)$$

Using FT, find the output y(t) to each of the following input signals.

i) $x(t)=e^{-t}u(t)$

ii) $x(t)=u(t)$ **(08 Marks)**

b. Find the Nyquist rate and Nyquist interval for each of the following signals:

i) $x(t)=\sin c^2(200t)$

ii) $x(t)=2\sin c(50t)\sin(5000\pi t)$ **(06 Marks)**

c. An LTI system is described by $H(f)=\frac{4}{2+j2\pi f}$ find its response y(t) if the input is $x(t)=u(t)$ **(06 Marks)**

7 a. Define ROC and list its properties. **(04 Marks)**

b. State and prove time reversal property of z-transform. **(04 Marks)**

c. Determine the inverse z-transform of $x(z)=\frac{1}{(1+z^{-1})(1-z^{-1})^2}$; ROC; $|z|>1$ **(06 Marks)**

d. Determine z-transform and ROC of $x(n)=\left(\frac{1}{3}\right)^n \sin\left(\frac{\pi}{4}n\right)u(n)$ **(06 Marks)**

8 a. A causal, stable discrete time system is defined by,

$$y(n)=\frac{5}{6}y(n-1)-\frac{1}{6}y(n-2)+x(n)-2x(n-1)$$

Determine

i) System function H(z) and magnitude response at zero frequency.

ii) Impulse response of the system.

iii) Output y(n) for $x(n)=\delta(n)-\frac{1}{3}\delta(n-1)$ **(12 Marks)**

b. Solve the following difference equation for the given initial conditions and input,

$$y(n)-\frac{1}{9}y(n-2)=x(n-1)$$

With $y(-1)=0$, $y(-2)=1$ and $x(n)=3u(n)$ **(08 Marks)**

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10EC45

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Fundamentals of HDL

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Explain the behavioral and dataflow style descriptions of VHDL, with the example of an half-adder. **(10 Marks)**
b. Compare VHDL and verilog. **(04 Marks)**
c. Explain structure of VHDL and verilog with an example. **(06 Marks)**

2 a. Write VHDL code for 2×2 bits combinational array multiplier (Dataflow style description). **(06 Marks)**
b. List the data types used in VHDL and verilog. **(04 Marks)**
c. Write a dataflow description in both VHDL and verilog, for a full adder with active high enable (en = 1). **(10 Marks)**

3 a. Distinguish between signal assignment and variable assignment statements in VHDL. Also, write VHDL program for behavioral description of D-latch using signal assignment and variable assignment statements, separately. **(10 Marks)**
b. Explain formats of for loop and while loop statements in both VHDL and verilog. **(06 Marks)**
c. Write VHDL code to calculate the factorial of positive integers. **(04 Marks)**

4 a. Write the structural description for full adder, using two half adders. **(06 Marks)**
b. Explain binding between two modules in verilog. **(04 Marks)**
c. Write VHDL structural description of 3-bits synchronous up counter using JK master slave flip-flops. **(10 Marks)**

PART – B

5 a. Give an example code for a procedure and a function. **(06 Marks)**
b. Write VHDL description of a full adder using procedure. **(08 Marks)**
c. Write a verilog code for converting a fraction binary to real using task. **(06 Marks)**

6 a. Why a mixed type description is needed? Write the VHDL code to find largest element in an array. **(10 Marks)**
b. Write a note on packages in VHDL. **(03 Marks)**
c. Write VHDL code for the addition of 5×5 matrices using a package. **(07 Marks)**

7 a. With a mixed language description of a full adder, explain the invoking of VHDL entity from a verilog module. **(10 Marks)**
b. Write the mixed language description of a JK master-slave flip-flop with clear input. **(10 Marks)**

8 a. What is meant by synthesis? List and explain steps involved in synthesis. **(07 Marks)**
b. Write VHDL or verilog code for the signal assignment statement y = 2 * a + 5 for an entity with one input a of 3-bits and one output y of 4-bits. Show the mapping of this signal assignment to gate level. **(10 Marks)**
c. Explain extraction of synthesis information from an entity. **(03 Marks)**

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10EC46

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Linear ICs and Applications

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Define the following terms as applied to op-amp and mention their typical values for IC741: i) CMRR; ii) Slew rate; iii) PSRR; iv) Input off set voltage. **(08 Marks)**

b. Sketch an op-amp direct coupled difference amplifier circuit. Explain the operation of the circuit and derive an equation for the output voltage. **(07 Marks)**

c. Design a direct coupled non inverting amplifier to amplify 100mV signal using IC741 to a level of 4 volts. **(05 Marks)**

2 a. With a neat circuit diagram, explain capacitor coupled voltage follower with relevant design steps. **(07 Marks)**

b. Explain how the upper cutoff frequency can be set for inverting amplifiers. **(07 Marks)**

c. Design a capacitor coupled inverting amplifier using a IC741. Op-Amp to have a voltage gain of 75 output voltage amplitude of 3 Volts and a signal frequency range of 20Hz to 12 kHz. The load resistance is 470Ω. **(06 Marks)**

3 a. Explain Miller effect compensation. **(08 Marks)**

b. List the precautions to be observed for Op-Amp stability. **(06 Marks)**

c. Using an LM108 Op-Amp, design an inverting amplifier to amplify a 100mV signal by a factor 3. Select suitable frequency compensation ($I_{B(max)}$ for LM108 is 2nA). **(06 Marks)**

4 a. With neat circuit diagram and waveforms at various points, explain the working of high input impedance full wave precision rectifier. **(10 Marks)**

b. Compare the performance of a differential input/output amplifier with that of a difference amplifier. **(04 Marks)**

c. Design a precision voltage source to get an output of 8.5V using 741 Op Amp. The supply voltage is ±15V and zener diode has a tolerance of ±5%. **(06 Marks)**

PART – B

5 a. With neat circuit diagram and waveforms, explain the working triangular/rectangular waveform generator with frequency and duty cycle control. **(10 Marks)**

b. With neat circuit diagram, explain basic log amplifier and derive an expression its o/p voltage. **(05 Marks)**

c. Using a 741 Op Amp with a supply of ±12V, design a phase shift oscillator to have an output frequency of 3.5kHz. **(05 Marks)**

6 a. With neat circuit diagram and waveforms, explain the circuit operation of an Op Amp inverting Schmit trigger circuit. **(08 Marks)**

b. With neat circuit diagram and waveforms, explain the operation of Op Amp astable multivibrator. **(06 Marks)**

c. Using a 741 Op Amp, design a record order active high pass filter for a cut off frequency of 4.5 kHz. **(06 Marks)**

7 a. List and explain the characteristics of three terminal IC regulators. **(06 Marks)**

b. Explain the principle of switching regulator. Mention its advantages. **(07 Marks)**

c. Design a voltage regulator using IC723 to get voltage output of. **(07 Marks)**

8 a. Explain 555 timer as monostable multivibrator with relevant circuit diagram, waveforms and expressions. **(08 Marks)**

b. With a neat diagram, explain the working of weighted resistor DAC. **(06 Marks)**

c. Draw the block diagram of PLL and explain its operation. **(06 Marks)**

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MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. If l, m, n are the direction cosines of a line then prove that $l^2 + m^2 + n^2 = 1$ **(06 Marks)**

b. Find angle between any two diagonals of a cube. **(07 Marks)**

c. Find angle between two lines whose direction cosines satisfy the equations, $l + m + n = 0$ and $2l + 2m - mn = 0$. **(07 Marks)**

2 a. With the usual notations derive the equation of the plane in the form $lx + my + nz = 0$. **(06 Marks)**

b. Find the equation of the plane through (1, 2, -1) and perpendicular to the planes $x + y - 2z = 5$ and $3x - y + 4z = 12$. **(07 Marks)**

c. Find the shortest distance between the lines,

$$\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1} \text{ and}$$

$$\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$$

(07 Marks)

3 a. Prove that $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{c} \cdot \vec{a}) - \vec{c}(\vec{a} \cdot \vec{b})$. **(06 Marks)**

b. Find the sine of angle between the vectors $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$. **(07 Marks)**

c. Show that the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + 4\hat{j} - \hat{k}$ are coplanar. **(07 Marks)**

4 a. Find the unit normal vector to the space curve $\vec{r} = 4\sin t\,\hat{i} + 4\cos t\,\hat{j} + 3t\,\hat{k}$. **(06 Marks)**

b. A particle moves along the curve $\vec{r} = \cos 2t\,\hat{i} + \sin 2t\,\hat{j} + t\,\hat{k}$. Find the velocity and acceleration at $t = \frac{\pi}{8}$ along $\sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j} + \hat{k}$. **(07 Marks)**

c. Find angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x = z^2 + y^2 - 3$ at (2, -1, 2) **(07 Marks)**

5 a. Find the directional derivative of x^2yz^3 at (1, 1, 1) in the direction of $\hat{i} + \hat{j} + 2\hat{k}$. **(06 Marks)**

b. If $\vec{F} = (x + y + 1)\hat{i} + \hat{j} - (x + y)\hat{k}$ then show that $\vec{F} \cdot \text{curl}\,\vec{F} = 0$ **(07 Marks)**

c. Show that the vector $\vec{F} = (3x^2 - 2yz)\hat{i} + (3y^2 - 2zx)\hat{j} + (3z^2 - 2xy)\hat{k}$ is irrotational. **(07 Marks)**

6 a. Prove that $L[\sin at] = \frac{a}{s^2 + a^2}$. **(05 Marks)**

b. Find $L[\sin t \sin 2t \sin 3t]$. **(05 Marks)**

c. Find $L[te^{-t} \sin 2t]$. **(05 Marks)**

d. Find $L\left[\frac{e^{at} - e^{bt}}{t}\right]$. **(05 Marks)**

7 a. If $L[f(t)] = \int_0^\infty e^{-st} f(t)dt$ then prove that $L[f''(t)] = s^2 L[f(t)] - sf(0) - f'(0)$. **(05 Marks)**

b. Find $L^{-1}\left[\frac{s+2}{s^2 - 4s + 13}\right]$. **(05 Marks)**

c. Find $L^{-1}\left[\frac{s+1}{(s-2)^3}\right]$. **(05 Marks)**

d. Find $L^{-1}\left[\log\left(\frac{s-a}{s-b}\right)\right]$. **(05 Marks)**

8 a. Using Laplace transform solve $y'' - 2y' + y = e^{2t}$ with $y(0) = 0$, $y'(0) = 1$. **(10 Marks)**

b. Using Laplace transform solve the simultaneous equation,

$$\frac{dx}{dt} + y = \sin t$$

$$\frac{dy}{dt} + x = \cos t$$

given that x(0)=1, y(0) = 0 **(10 Marks)**

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